

# First theorems of Propositional Calculus

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## Abstract

This module includes first proofs of propositional calculus theorems.

## Specification

This document has the following specification:

Name:	luktheo1
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Rule version:	1.00.00
Origin:	<a href="http://www.qedeq.org/0_00_53/luktheo1_1.00.00_1.00.00.qedeq">http://www.qedeq.org/0_00_53/luktheo1_1.00.00_1.00.00.qedeq</a>

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## References

This document uses the results of the following documents:

Name:	lukaxiom
Version:	1.00.00
Rule version:	1.00.00
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pdf:	<a href="http://www.qedeq.org/lukaxiom_1.00.00_1.00.00.pdf">lukaxiom_1.00.00_1.00.00.pdf</a>

## Content

First we prove a theorem known as *identity*:

**Theorem 0.1 (theorem1).**

$$P \rightarrow P$$

*Proof.*

1	$P \rightarrow (Q \rightarrow P)$	add axiom axiom1
2	$P \rightarrow ((P \rightarrow P) \rightarrow P)$	replace $Q$ by $P \rightarrow P$ in 1
3	$(P \rightarrow (Q \rightarrow A)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow A))$	add axiom axiom2
4	$(P \rightarrow ((P \rightarrow P) \rightarrow A)) \rightarrow ((P \rightarrow (P \rightarrow P)) \rightarrow (P \rightarrow A))$	replace $Q$ by $P \rightarrow P$ in 3
5	$(P \rightarrow ((P \rightarrow P) \rightarrow P)) \rightarrow ((P \rightarrow (P \rightarrow P)) \rightarrow (P \rightarrow P))$	replace $A$ by $P$ in 4
6	$(P \rightarrow (P \rightarrow P)) \rightarrow (P \rightarrow P)$	MP with 2, 5
7	$P \rightarrow (P \rightarrow P)$	replace $Q$ by $P$ in 1
8	$P \rightarrow P$	MP with 7, 6

□

Now we prove a theorem known as (one form of) *hypothetical syllogism*:

**Theorem 0.2 (theorem2).**

$$(Q \rightarrow A) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow A))$$

*Proof.*

1	$P \rightarrow (Q \rightarrow P)$	add axiom axiom1
2	$P \rightarrow ((Q \rightarrow A) \rightarrow P)$	replace $Q$ by $Q \rightarrow A$ in 1
3	$((P \rightarrow (Q \rightarrow A)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow A))) \rightarrow ((Q \rightarrow A) \rightarrow ((P \rightarrow (Q \rightarrow A)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow A))))$	replace $P$ by $(P \rightarrow (Q \rightarrow A)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow A))$ in 2
4	$(P \rightarrow (Q \rightarrow A)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow A))$	add axiom axiom2
5	$(Q \rightarrow A) \rightarrow ((P \rightarrow (Q \rightarrow A)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow A)))$	MP with 4, 3
6	$(B \rightarrow (Q \rightarrow A)) \rightarrow ((B \rightarrow Q) \rightarrow (B \rightarrow A))$	replace $P$ by $B$ in 4
7	$(B \rightarrow (C \rightarrow A)) \rightarrow ((B \rightarrow C) \rightarrow (B \rightarrow A))$	replace $Q$ by $C$ in 6
8	$(B \rightarrow (C \rightarrow D)) \rightarrow ((B \rightarrow C) \rightarrow (B \rightarrow D))$	replace $A$ by $D$ in 7
9	$((Q \rightarrow A) \rightarrow (C \rightarrow D)) \rightarrow (((Q \rightarrow A) \rightarrow C) \rightarrow ((Q \rightarrow A) \rightarrow D))$	replace $B$ by $Q \rightarrow A$ in 8
10	$((Q \rightarrow A) \rightarrow ((P \rightarrow (Q \rightarrow A)) \rightarrow D)) \rightarrow (((Q \rightarrow A) \rightarrow (P \rightarrow (Q \rightarrow A))) \rightarrow ((Q \rightarrow A) \rightarrow D))$	replace $C$ by $(P \rightarrow (Q \rightarrow A)) \rightarrow D$ in 9
11	$((Q \rightarrow A) \rightarrow ((P \rightarrow (Q \rightarrow A)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow A)))) \rightarrow (((Q \rightarrow A) \rightarrow (P \rightarrow (Q \rightarrow A))) \rightarrow ((Q \rightarrow A) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow A))))$	replace $D$ by $(P \rightarrow Q) \rightarrow (P \rightarrow A)$ in 10
12	$((Q \rightarrow A) \rightarrow (P \rightarrow (Q \rightarrow A))) \rightarrow ((Q \rightarrow A) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow A)))$	MP with 5, 11
13	$P \rightarrow (B \rightarrow P)$	replace $Q$ by $B$ in 1
14	$(Q \rightarrow A) \rightarrow (B \rightarrow (Q \rightarrow A))$	replace $P$ by $Q \rightarrow A$ in 13
15	$(Q \rightarrow A) \rightarrow (P \rightarrow (Q \rightarrow A))$	replace $B$ by $P$ in 14
16	$(Q \rightarrow A) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow A))$	MP with 15, 12

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